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# The Entropy of 4D Black Holes and the Enhançon

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## ABSTRACT

We consider the physics of enhançons as applied to four dimensional black holes which are constructed by wrapping both D-branes and NS-branes on K3. As was recently shown for the five dimensional black holes, the enhançon is crucial in maintaining consistency with the second law of thermodynamics. This is true for both the D-brane *and* NS-brane sectors of these black holes. In particular NS5-branes in both type IIA and IIB string theory are found to exhibit enhançon physics when wrapped on a K3 manifold.

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# 1 Introduction

One of the most fruitful applications of D-branes[1] in string theory has been the study of black holes. Most notably [2] D-brane technology has been used to give a statistical mechanical description of the Bekenstein-Hawking entropy [3] of a large class of five [4, 5, 6, 7] and four [8, 9, 10, 11] dimensional black holes — see refs. [12, 13, 14, 15, 16, 17] for extensive references and reviews.

Essential to the embedding of charged black holes in string theory is the notion of compactification. In general one begins by constructing a ten-dimensional solution of type II supergravity which contains the desired charges. This solution is then dimensionally reduced on an appropriate compact manifold so that the lower dimensional solution has an event horizon with finite area and thus a non-trivial entropy. A fundamental element of these constructions is that all of the stringy constituents are wrapped around some part of the compact manifold so that the final configuration appears as a point like source in the lower dimensional space.

It has been known for some time [18, 19] that wrapping a Dp-brane around certain manifolds with non-trivial topology induces a charge associated with a D(p-4)-brane onto the world volume of the original Dp-brane. In particular when the compactification manifold is K3 a unit of negative charge is induced leading to a negative contribution to the total tension of the wrapped brane [20]. When large numbers of wrapped branes are brought together so that there is a non-trivial back reaction on the geometry these facts have interesting consequences. As was pointed out in ref. [21] these situations generically give rise to regions of space time where wrapped Dp-brane probes of the geometry acquire a negative tension. These regions also contain naked time-like singularities known as repulsons[22, 23, 24]. It was proposed in ref. [21] that these regions are in fact unphysical and should be excised from the space time altogether. Inside of the incision surface there is simply flat space. Dp-brane probes become tensionless on the (outer) boundary of the excised region and cannot proceed further into the space time. Finally inside of the boundary locus there is an enhanced gauge symmetry; hence the name *enhanc on*. The consistency of this proposal from the point of view of supergravity has recently been studied in ref. [25].

In this paper we will consider a particular class of four dimensional charged black holes in type IIB string theory. The *asymptotic* charges correspond to  $Q_1$  D1-branes embedded in the world volume of  $Q_5$  D5-branes and  $Q_{F1}$  fundamental (F1-) strings inside  $Q_{NS}$  NS5-branes. The reduction to four dimensions is accomplished by compactifying on  $K3 \times T^2$  where both the D5-branes and the NS5-branes are wrapped around K3 leaving a pair of effective strings wrapping orthogonal cycles on the torus — see below. This black hole may be obtained from the more familiar D2/D6/NS5/P black hole considered in ref. [26, 9, 12] by acting with T-duality on the circle around which the momentum modes propagate.

There are several novel features of this particular black hole. First, up to the interchange of cycles on the torus this configuration is self dual under type IIB S-duality. Also, since this configuration involves orthogonal NS5-branes and D5-branes it might be expected that for finite string coupling the physics of  $(p, q)$  webs [27] will be important in understanding the dynamics of these black holes. Further, due to the BPS property of this system, the two effective strings are independent and hence, as will be seen below, there are two independent *enhanc on* effects.

Since this black hole is obtained by wrapping D5-branes on K3 it is expected that the enhançon mechanism will be relevant. Indeed it was recently shown in ref. [28] that for the case of five dimensional black holes involving D5-branes wrapped on K3 the enhançon mechanism plays a decisive role in ensuring that the string theory embedding of these black holes is consistent with the second law of thermodynamics. The same result is found here: Enhançon physics arises to prevent, in all cases, D5-branes from being absorbed into the black hole when their absorption would serve to decrease the entropy.

In addition to the D5-branes the NS5-branes are also wrapped around K3 and might therefore be expected to exhibit enhançon physics. Recall that under S-duality D5-branes are exchanged with NS5-branes and D1-branes are interchanged with F1-strings. Further, the enhanced gauge symmetry found inside of the enhançon locus should survive the journey to strong coupling and should therefore be present in any S-dual formulation. As we shall see there is indeed an enhançon effect for wrapped NS5-branes. In the context of four dimensional black holes the role of the NS5-brane enhançon is to prevent violations of the second law of thermodynamics.

The remainder of this paper is organized as follows. Section 2 contains a discussion of these four dimensional black holes and the enhançon from the point of view of supergravity. These results are then reproduced via probe calculations in section 3. Section 4 is a discussion of the implications of enhançon physics for the second law of thermodynamics as it pertains to the black holes considered here. Motivated by the relevance of enhançon physics for NS5-branes within black holes, section 5 focuses on enhançons for both type IIA/B NS5-branes in isolation. In section 6 conclusions are presented and some interesting open problems are discussed.

## 2 Black Holes in Four Dimensions and the Enhançon

The four dimensional black hole to be considered in this paper is constructed from the orthogonal intersection of a D1/D5 bound state and an F1/NS5 bound state. The D1-branes are aligned along the  $x^4$  direction, the D5-branes fill the directions spanned by  $(x^4, x^5, x^6, x^7, x^8)$ , the NS5-branes fill  $(x^5, x^6, x^7, x^8, x^9)$  and the F1-strings lie along  $x^9$ . The reduction to four dimensions is accomplished by wrapping  $(x^5, x^6, x^7, x^8)$  on a K3 manifold leaving an effective D1/D5 string in the  $x^4$  direction and an effective F1/NS5 string in the  $x^9$  direction. The effective strings are then each compactified on circles of radii  $R_4$  and  $R_9$  respectively, leaving a black hole in  $(t, x^1, x^2, x^3)$ .

The ten dimensional Einstein frame metric for this D1/D5/F1/NS5 system is,

$$ds^2 = -h_1^{-3/4}h_5^{-1/4}h_{F1}^{-3/4}h_{NS}^{-1/4}dt^2 + h_1^{-3/4}h_5^{-1/4}h_{F1}^{1/4}h_{NS}^{3/4}dx_4^2 + h_1^{1/4}h_5^{3/4}h_{F1}^{-3/4}h_{NS}^{-1/4}dx_9^2 + h_1^{1/4}h_5^{-1/4}h_{F1}^{1/4}h_{NS}^{-1/4}ds_{K3}^2 + h_1^{1/4}h_5^{3/4}h_{F1}^{1/4}h_{NS}^{3/4}(dr^2 + r^2d\Omega_2^2). \quad (1)$$

The dilaton, Ramond-Ramond and Kalb-Ramond potentials for this solution are,

$$\begin{aligned} e^{2\phi} &= \frac{h_1 h_{NS}}{h_5 h_{F1}} \\ C_{(2)} &= h_1^{-1}dt \wedge dx^4 \\ C_{(6)} &= h_5^{-1}dt \wedge dx^4 \wedge \cdots \wedge dx^8 \end{aligned}$$

$$\begin{aligned}
B_{(2)} &= h_{F1}^{-1} dt \wedge dx^9 \\
B_{(6)} &= h_{NS}^{-1} dt \wedge dx^5 \wedge \cdots \wedge dx^9
\end{aligned} \tag{2}$$

where  $B_{(6)}$  is the six form which couples electrically to the NS5-brane. The harmonic functions in all of the above expressions are given by,

$$\begin{aligned}
h_1 &= 1 + \frac{c_1 Q_1}{r} & h_5 &= 1 + \frac{c_5 Q_5}{r} \\
h_{F1} &= 1 + \frac{c_{F1} Q_{F1}}{r} & h_{NS} &= 1 + \frac{c_{NS} Q_{NS}}{r}
\end{aligned} \tag{3}$$

and in four dimensions the constants  $c_i$  are [12],

$$c_1 = \frac{g l_s^2}{2R_9} \frac{V^*}{V} \quad c_5 = \frac{g l_s^2}{2R_9} \quad c_{F1} = \frac{g^2 l_s^2}{2R_4} \frac{V^*}{V} \quad c_{NS} = \frac{l_s^2}{2R_4} \tag{4}$$

Notice that the local string frame volume of the K3 manifold is given by,

$$V(r) = \frac{h_1}{h_5} V \tag{5}$$

where  $V$  is the asymptotic volume of the K3. For  $c_1 Q_1 < c_5 Q_5$  this volume is shrinking monotonically as we approach  $r = 0$ . Specifically, we have  $V(0) < V$  and so in accord with the results of [21] we may expect that enhançon physics will be important at some finite value of  $r$ . Note that  $V(r)$  is unaware of the F1/NS5-brane component of the geometry and so we expect that when  $V(r) = V^*$  there will be an enhançon locus beyond which individual D5-branes may not proceed in a supersymmetric manner. Despite the fact that the local volume of K3 is independent of the wrapped NS5-branes it will be found below that there is also a shell located at a finite radius which NS5-branes are forbidden from passing.

As a final comment notice that the local value of the string coupling at  $r = 0$  is given by

$$g^2 e^{2\phi} = \frac{Q_1}{Q_5} \frac{Q_{NS}}{Q_{F1}} \tag{6}$$

and is small provided that,

$$\frac{Q_1}{Q_5} \ll \frac{Q_{F1}}{Q_{NS}} \tag{7}$$

in which case string loops may safely be neglected.

Upon reducing to four dimensions the surface at  $r = 0$  is an extremal horizon with vanishing surface gravity and non-vanishing area which gives a Bekenstein-Hawking entropy,

$$S_{BH} = 2\pi \sqrt{Q_1 Q_5 Q_{F1} Q_{NS}} \tag{8}$$

Since this black hole is constructed from D5-branes wrapped on a K3 surface one expects that the enhançon physics uncovered in ref. [28] will be relevant here. However, as remarked in the introduction, S-duality suggests that wrapping an NS5-brane on a K3 manifold will induce negative amounts of F1-string charge and therefore it seems that enhançon physics will also be relevant for this part of the system. This will be demonstrated later via an explicit probe

calculation but for now it will simply be assumed and its consequences explored. With this in mind, recall that the integers  $Q_1, Q_5, Q_{F1}, Q_{NS}$  measure the asymptotic charges associated with the constituents of the black hole. In order to facilitate the following analysis it is convenient to introduce another set of integers,  $N_1, N_5, N_{F1}, N_{NS}$ , which count the actual number of branes in the system. In particular  $Q_1 = N_1 - N_5$  and  $Q_{F1} = N_{F1} - N_{NS}$  while  $N_5 = Q_5$  and  $N_{NS} = Q_{NS}$ . In terms of these parameters the entropy in eqn. (8) becomes,

$$S_{BH} = 2\pi \sqrt{(N_1 - N_5)(N_{F1} - N_{NS})N_5 N_{NS}}. \quad (9)$$

Writing the entropy in this way suggests that this class of black holes is, for certain ranges of parameters, at odds with the second law of thermodynamics. To see this assume that  $N_1 = 2N_5$  and then consider slowly (*i.e.*, adiabatically) moving a single D5-brane from infinity into the black hole. It is not difficult to see that this will cause a decrease in the total entropy. Of course the same statement can be made about the F1/NS5 sector of this black hole. See section 4 below for a complete discussion.

For the case of five dimensional black holes constructed from D1-branes, D5-branes and momentum modes it was shown in ref. [28] that the resolution of this paradox lies in the physics of the enhançon mechanism [21]. In the following it will be shown that the class of black holes under consideration in fact require *two* enhançons to maintain consistency with the second law.

The physics pertaining to the D1/D5 sector of this black hole is already well known [21, 28]. Since we expect that there exists a radius inside of which D5-branes cannot penetrate, the supergravity solution must be modified in the interior. To model this, while at the same time remaining completely general, it is assumed that some number of the D-branes *i.e.*,  $\delta N_5$  D5-branes and  $\delta N_1$  D1-branes, are evenly distributed over a two sphere located at a radius denoted by  $r_c$ . The black hole in the interior therefore consists of  $N'_5 = N_5 - \delta N_5$  D5-branes and  $N'_1 = N_1 - \delta N_1$  D1-branes corresponding to asymptotic charges  $Q'_5 = N'_5$  and  $Q'_1 = N'_1 - N'_5$ .

In order to understand better the F1/NS5 sector of this black hole we also postulate that there exists a radius given by  $r_b$  at which a shell composed of  $\delta N_{NS}$  NS5-branes and  $\delta N_{F1}$  F1-strings resides. The black hole inside of this radius (and inside of  $r_c$ ) therefore contains  $N'_{NS} = N_{NS} - \delta N_{NS}$  NS5-branes and  $N'_{F1} = N_{F1} - \delta N_{F1}$  fundamental strings which corresponds to asymptotic charges  $Q'_{NS} = N'_{NS}$  and  $Q'_{F1} = N'_{F1} - N'_{NS}$ .

It is important to note that if there is no enhançon effect for NS5-branes the shell located at  $r = r_b$  is completely benign and there is no reason that the F1/NS5-branes located there cannot be placed inside of the black hole. It will be shown however that enhançon physics is relevant and there is a radius inside of which NS5-branes may not proceed unless suitable conditions, which will be derived below, are satisfied.

As a last point, note that the relative magnitudes of  $r_c$  and  $r_b$  are, for now, unimportant because of the BPS property and the fact that the curvature of K3 does not induce any F1-string charge on the D5-branes nor any D1-brane charge on the NS5-branes. In effect the two shells are independent.

## 2.1 D1/D5-brane Enhancements

The more familiar case of the D1/D5 enhancement is considered first. Inside the radius  $r_c$  the supergravity solution in eqn. (1) must be modified to account for the different charges. This new solution must however match onto the exterior solution in a smooth way. This is achieved by simply replacing the harmonic functions related to the D1-branes and D5-branes in (1) by the following ‘hatted’ functions,

$$\hat{h}_1 = 1 + \frac{c_1 Q'_1}{r} + \frac{c_1(Q_1 - Q'_1)}{r_c} \quad \hat{h}_5 = 1 + \frac{c_5 Q'_5}{r} + \frac{c_5(Q_5 - Q'_5)}{r_c}. \quad (10)$$

The NS5-brane and F1-string harmonic functions are unchanged and do not figure in the enhancement effect for the D1/D5 system. Even though this patched metric is continuous at  $r = r_c$  (the ‘hatted’ functions were chosen to ensure this) there will be a discontinuity in the extrinsic curvature of the gluing surface  $r = r_c$  [29]. This has an interpretation as the stress energy of the D1/D5-brane shell—for a complete discussion see ref. [25]. The extrinsic curvature of the  $r = r_c$  surface is,

$$K_{AB}^\pm = \frac{1}{2} n_\pm^C \partial_C g_{AB}|_{r=r_c} = \mp \frac{1}{2} \sigma \partial_r g_{AB}|_{r=r_c} \quad (11)$$

where  $n_\pm = \mp \sigma \partial_r$  is the outward directed unit normal vector with  $\sigma = 1/\sqrt{g_{rr}}|_{r=r_c}$ . The discontinuity of the extrinsic curvature across the surface  $r = r_c$  is defined to be  $\gamma_{AB} = K_{AB}^+ + K_{AB}^-$  and the surface stress tensor is,

$$S_{AB} = \frac{1}{8\pi G} (\gamma_{AB} - g_{AB} \gamma^C{}_C) \quad (12)$$

where  $16\pi G = (2\pi)^7 g^2 l_s^8$  is the ten dimensional Newton’s constant.

Calculating the stress tensor of the surface at  $r = r_c$  one finds,

$$\begin{aligned} 8\pi G S_{tt} &= \frac{\sigma}{2} \left( \frac{h'_5}{h_5} + \frac{h'_1}{h_1} - \frac{\hat{h}'_5}{\hat{h}_5} - \frac{\hat{h}'_1}{\hat{h}_1} \right) g_{tt} \\ 8\pi G S_{44} &= \frac{\sigma}{2} \left( \frac{h'_5}{h_5} + \frac{h'_1}{h_1} - \frac{\hat{h}'_5}{\hat{h}_5} - \frac{\hat{h}'_1}{\hat{h}_1} \right) g_{44} \\ 8\pi G S_{99} &= 0 \\ 8\pi G S_{ab} &= \frac{\sigma}{2} \left( \frac{h'_5}{h_5} - \frac{\hat{h}'_5}{\hat{h}_5} \right) g_{ab} \\ 8\pi G S_{ij} &= 0 \end{aligned} \quad (13)$$

where the indices  $a, b$  run over the K3 and  $i, j$  denote the angular coordinates on the two-sphere at the incision point  $r_c$ . This result is exactly as expected. The D5-branes do not wrap the  $x^9$  circle so there is no stress-energy in this direction. Also, since the D1-branes do not wrap the K3 manifold the stress tensor in these directions is independent of  $h_1$  and  $\hat{h}_1$ . Finally, since this is a BPS configuration, there is no stress-energy associated with placing the D5-branes and D1-branes on the shell at  $r = r_c$ . This result is also manifestly independent of the NS5-branes and fundamental strings.

The energy density, or tension, of the effective string in the  $x^4$  direction is given by,

$$8\pi GT = \frac{\sigma}{2} \left( \frac{\hat{h}'_1}{\hat{h}_1} + \frac{\hat{h}'_5}{\hat{h}_5} - \frac{h'_1}{h_1} - \frac{h'_5}{h_5} \right) \quad (14)$$

This expression can be rewritten as,

$$T = \frac{h_1^{-1/4} h_5^{1/4} h_{F1}^{1/4} h_{NS}^{-1/4} (Q_1 - Q'_1) \tau_1}{A_{S^2} A_{K3} A_{S^1_9}} + \frac{h_1^{1/4} h_5^{-1/4} h_{F1}^{-1/4} h_{NS}^{1/4} (Q_5 - Q'_5) \tau_5}{A_{S^2} A_{S^1_9}} \quad (15)$$

where  $\tau_{1,5}$  are the canonical flat space tensions of D1-branes and D5-branes respectively. Here  $A_{K3}, A_{S^2}, A_{S^1_9}$  are the proper areas, as measured at  $r = r_c$  in the Einstein frame, of the K3 manifold,  $S^2$  and the  $x^9$  circle respectively. These factors appear in the formula for the tension since, from the supergravity point of view, the branes involved have been smeared over these directions and hence the tension appears as an average over these directions. Also,  $Q_1 - Q'_1 = \delta N_1 - \delta N_5$  and  $Q_5 - Q'_5 = \delta N_5$  are the effective number of D1-branes and D5-branes residing on the shell. In order to understand this result one must recall that the calculations presented here have been carried out using the Einstein frame metric. In string frame the *local* value of the tension of a D1-brane, for example, can be obtained by varying the the Born-Infeld action,

$$S = \tau_1 \int_{WV} e^{-\phi} \sqrt{P[g_{AB}^{string}]}. \quad (16)$$

with respect to the string frame metric. The result is  $T_1 = e^{-\phi} \tau_1$ . Switching to Einstein frame using  $g_{AB}^{string} = e^{\phi/2} g_{AB}^{Einstein}$  and varying with respect to the Einstein frame metric one finds that the *local* value of the D1-brane tension is,

$$T_1^{Einstein} = e^{-\phi/2} \tau_1 = h_1^{-1/4} h_5^{1/4} h_{F1}^{1/4} h_{NS}^{-1/4} \tau_1 \quad (17)$$

which is precisely the result obtained in eqn. (15) above. Similar considerations for the D5-brane verify that eqn. (15) is the expected formula for the effective tension in the string.

Substituting the harmonic functions of eqns. (3) and (10) into eqns. (14) or (15) it is found that the tension of the effective string remains *non-negative* so long as,

$$r_c \geq \frac{c_5 c_1}{\delta N_5 (c_5 - c_1) - \delta N_1 c_1} (\delta N_5 (2N_5 - N_1) - \delta N_1 N_5) \quad (18)$$

Further if there are no D1-branes on the shell, *i.e.*,  $\delta N_1 = 0$ , then one finds that,

$$\begin{aligned} r_c &\geq \frac{c_5 c_1}{c_5 - c_1} (2N_5 - N_1) \\ &= \frac{gl_s^2}{2R_9} \frac{(2N_5 - N_1)}{V/V^* - 1} \equiv r_e \end{aligned} \quad (19)$$

which is precisely the enhançon radius originally found in ref. [21] rewritten in terms of the four dimensional parameters given in eqn. (4).

From eqn. (19) it is obvious that for  $2N_5 > N_1$  the tension of the D5-brane shell vanishes outside of the horizon, *i.e.*,  $r_e > 0$ . Inside of this radius the tension would be negative and as emphasized in ref. [21] supersymmetry would be broken. Therefore all of the D5-branes on the

shell must remain outside of the black hole. In particular they cannot contribute to the entropy. Note, however, that for  $\delta N_1 > 0$  we can have  $r_c < r_e$  and still maintain non-negative tension in the shell. Hence when explicit D1-branes are included on the shell it can be brought closer to the horizon. Each additional D1-brane on the shell brings the minimum radius infinitesimally closer to  $r = 0$ . The enhançon thus belies an onion like structure. This will be seen in more detail in the following section where it will be shown that probes which are made up of both D1- and D5-branes can move closer to the horizon, while maintaining positive tension, than D5-branes alone. As we shall see in some cases such combined probes may even reach  $r = 0$ .

Finally, it is straight forward to check that the string frame volume of the K3 defined in eqn. (5) satisfies  $V(r_e) = V^*$ .

## 2.2 F1/NS5-brane Enhançons

Next, consider the shell of F1/NS5-branes sitting at  $r = r_b$ . Inside of this radius the metric is matched onto the metric of eqn. (1) where the harmonic functions of eqn. (3) are used for the D1/D5-branes<sup>1</sup> and those for the NS5-branes and F1-strings are given by,

$$\hat{h}_{F1} = 1 + \frac{c_{F1}Q'_{F1}}{r} + \frac{c_{F1}(Q_{F1} - Q'_{F1})}{r_b} \quad \hat{h}_{NS} = 1 + \frac{c_{NS}Q'_{NS}}{r} + \frac{c_{NS}(Q_{NS} - Q'_{NS})}{r_b} \quad (20)$$

Again, the patched metric is continuous but there is a discontinuity in the extrinsic curvature. Calculating the stress tensor at the surface  $r = r_b$  yields,

$$\begin{aligned} 8\pi GS_{tt} &= \frac{\sigma}{2} \left( \frac{h'_{NS}}{h_{NS}} + \frac{h'_{F1}}{h_{F1}} - \frac{\hat{h}'_{NS}}{\hat{h}_{NS}} - \frac{\hat{h}'_{F1}}{\hat{h}_{F1}} \right) g_{tt} \\ 8\pi GS_{44} &= 0 \\ 8\pi GS_{99} &= \frac{\sigma}{2} \left( \frac{h'_{NS}}{h_{NS}} + \frac{h'_{F1}}{h_{F1}} - \frac{\hat{h}'_{NS}}{\hat{h}_{NS}} - \frac{\hat{h}'_{F1}}{\hat{h}_{F1}} \right) g_{44} \\ 8\pi GS_{ab} &= \frac{\sigma}{2} \left( \frac{h'_{NS}}{h_{NS}} - \frac{\hat{h}'_{NS}}{\hat{h}_{NS}} \right) g_{ab} \\ 8\pi GS_{ij} &= 0 \end{aligned} \quad (21)$$

which, as before is exactly the expected result. In this case there are no branes wrapping the  $x^4$  circle, there are no F1-strings wrapping the K3 and again the BPS property of the shell is indicated by the vanishing of the stress tensor in the sphere directions. The tension of the effective string lying along  $x^9$  is,

$$T = \frac{\sigma}{2} \left( \frac{\hat{h}'_{F1}}{\hat{h}_{F1}} + \frac{\hat{h}'_{NS}}{\hat{h}_{NS}} - \frac{h'_{F1}}{h_{F1}} - \frac{h'_{NS}}{h_{NS}} \right) \quad (22)$$

as in the previous example this can be rewritten as,

$$T = \frac{h_1^{1/4} h_5^{-1/4} h_{F1}^{-1/4} h_{NS}^{1/4} (Q_{F1} - Q'_{F1}) \tau_{F1}}{A_{S_4^1} A_{S^2} A_{K3}} + \frac{h_1^{-1/4} h_5^{1/4} h_{F1}^{1/4} h_{NS}^{-1/4} (Q_{NS} - Q'_{NS}) \tau_{NS}}{A_{S^2} A_{S_4^1}} \quad (23)$$

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<sup>1</sup>In the case that  $r_b < r_c$  one may wish to use the harmonic functions given in eqn. (10) to represent the D1/D5-brane charges in the black hole. The difference is inconsequential since these harmonic functions do not change form as one crosses the surface  $r = r_b$  and hence whichever set of harmonic functions are chosen will drop out of the stress tensor calculation.



where  $\tau_{F1,NS}$  are the canonical flat space tensions of F1-strings and NS5-branes respectively. Here  $A_{K3}, A_{S^2}$ , are the proper areas introduced above and  $A_{S^4}$  is the proper area of the  $x^4$  circle. All of these are now measured at  $r = r_b$  in the Einstein frame. Just as before these factors appear in the formula for the tension because the branes involved have been smeared over these directions and our calculations have been performed using the Einstein frame metric. In particular varying the world volume action for F1-strings or NS5-branes with respect to the Einstein frame metric yields precisely the local value of the tensions found in eqn. (23) — see section 2.1 above. Also,  $Q_{F1} - Q'_{F1} = \delta N_{F1} - \delta N_{NS}$  and  $Q_{NS} - Q'_{NS} = \delta N_{NS}$  are the effective numbers of F1-strings and NS5-branes residing on the shell. Positivity of this tension requires,

$$r_b \geq \frac{c_{NS}c_{F1}}{\delta N_{NS}(c_{NS} - c_{F1}) - \delta N_{F1}c_{F1}} (\delta N_{NS}(2N_{NS} - N_{F1}) - \delta N_{F1}N_{NS}) \quad (24)$$

which for  $\delta N_{F1} = 0$  can be evaluated as,

$$\begin{aligned} r_b &\geq \frac{c_{NS}c_{F1}}{c_{NS} - c_{F1}} (2N_{NS} - N_{F1}) \\ &= \frac{g^2 l_s^2}{2R_4} \frac{(2N_{NS} - N_{F1})}{V/V^* - g^2} \equiv \tilde{r}_e \end{aligned} \quad (25)$$

Again the tension in the shell vanishes outside of the horizon for  $2N_{NS} > N_{F1}$  and moving individual NS5-branes off the shell towards the horizon will lead to negative tensions and unphysical results. As in the previous section there is an onion structure since adding explicit F1-strings to the shell allows it to be moved closer to the horizon while maintaining supersymmetry.

Since in refs. [21, 25, 28] tensionless shells were found to coincide precisely with tensionless supersymmetric probes of the geometry giving rise to the enhançon effect one should interpret the results here as indicating the presence of an enhançon effect for NS5-branes. Further evidence for NS5-brane enhançons will be presented in the following section.

It is interesting to note that the form of eqn. (18) is identical to that of eqn. (24) up to the replacement of D1/D5 labels with those of F1/NS5. This is exactly the prescription given by S-duality<sup>2</sup> and thus these two radii, as well as the two effective strings, are interchanged under type IIB S-duality. More will be said about this, as well as the implications of T-duality, in the following sections.

As a final comment, observe that in eqn. (25) there is an additional factor of the asymptotic string coupling  $g$  relative to eqn. (19). Thus (for weak coupling) the enhançon shell for NS5-branes sits at a radius which is parametrically smaller than that for the D5-brane.

### 3 Probes and Enhançons

In this section the black hole discussed above from the point of view of pure supergravity will be analyzed from a more stringy perspective by probing the black hole geometry using D1/D5/F1/NS5-branes. These are all natural probes to use since, when aligned correctly, they do not break any additional supersymmetry. Of course the probe actions for D1-branes, D5-branes and fundamental strings are all well known, however, the action for the NS5-brane is more complicated and it will be obtained here by considering S-duality of the D5-brane action.

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<sup>2</sup>Recall that under S-duality  $g \rightarrow 1/g$  and  $l_s^2 \rightarrow gl_s^2$ .

### 3.1 D5-brane Probes and S-Duality

To begin, consider the probe Born-Infeld action for an unwrapped D5-brane [30],

$$S_{D5} = -\tau_{D5} \int d^6 y e^{-\phi} \sqrt{-\det P[g_{AB}^{string}]} \quad (26)$$

where  $\tau_{D5} = (2\pi)^{-5} g^{-1} l_s^{-6}$  and  $P[g_{AB}^{string}]$  is the pull-back of the background string frame metric to the six dimensional world volume. In order to act with S-duality on this action it is convenient to work with the Einstein frame metric, which is invariant under S-duality and defined by,  $g_{AB}^{Einstein} = e^{-\phi/2} g_{AB}^{string}$ . The only field which transforms under S-duality is then the dilaton,  $\phi \rightarrow -\phi$ . The result after transforming back to string frame is,

$$S_{NS} = -\tau_{NS} \int d^6 y e^{-2\phi} \sqrt{-\det P[g_{AB}^{string}]} \quad (27)$$

where  $\tau_{NS} = \tau_{D5}/g$ . In order to determine the probe action for a wrapped NS5-brane we recall the analogous action for a D5-brane wrapped on K3 [18, 19, 20, 21],

$$S_{D5} = - \int d^2 \xi e^{-\phi} (\tau_5 V(r) - \tau_{D1}) \sqrt{-\det P[g_{AB}^{string}]} + \tau_{D5} \int_{K3 \times M} C_{(6)} - \tau_{D1} \int_M C_{(2)} \quad (28)$$

Considering that S-duality interchanges D1-branes and fundamental strings and recalling that the probe action for a fundamental string is simply given by the Nambu-Goto action we can now write down the probe action for an NS5-brane wrapped on a K3 surface and use this to probe the metric of the F1/NS5 bound state given in eqn. (48). The probe action is,

$$S_{NS} = - \int d^2 \zeta (\tau_{NS} e^{-2\phi} V(r) - \tau_{F1}) \sqrt{-\det P[g_{AB}^{string}]} + \tau_{NS} \int_{K3 \times M} B_{(6)} - \tau_{F1} \int_M B_{(2)} \quad (29)$$

This has been written down on physical grounds dictated by S-duality and supersymmetry. In this expression  $\tau_{F1} = (2\pi l_s^2)^{-1}$ ,  $M$  represents the unwrapped directions of the NS5-brane's world volume and  $V(r)$  is the local volume of the K3 surface. Note that standard supergravity conventions for the tensions and charges, as discussed in ref. [31], have been employed in writing down these actions.

### 3.2 Probing the Black Hole

In this section the probe technology above will be applied to the four dimensional black hole discussed previously. This analysis is most easily performed using the string frame metric for this black hole which is given by,

$$\begin{aligned} ds_{string}^2 &= -h_1^{-1/2} h_5^{-1/2} h_{F1}^{-1} dt^2 + h_1^{-1/2} h_5^{-1/2} h_{NS} dx_4^2 + h_1^{1/2} h_5^{1/2} h_{F1}^{-1} dx_9^2 \\ &+ h_1^{1/2} h_5^{-1/2} ds_{K3}^2 + h_1^{1/2} h_5^{1/2} h_{NS} (dr^2 + r^2 d\Omega_2^2) \end{aligned} \quad (30)$$

while the remainder of the closed string fields are the same as in eqn. (2). We will probe the metric given in eqn. (30) using all of the constituents of the black hole. In order to make a direct comparison with the supergravity calculations above the probes used in this section will be made up of, either D1/D5 bound states involving  $n_5$  D5-branes and  $n_1$  D1-branes or

F1/NS5 bound states composed of  $n_{NS}$  NS5-branes and  $n_{F1}$  fundamental strings. It should be emphasized that these will be considered as true bound states in the sense that the D1-branes exist inside the world volume of the D5-branes as finite size instantons. This implies that the strings which describe the separation of the D1-branes from the D5-branes in the Coulomb phase are all massive. Similar comments apply for the F1-strings embedded into the NS5-brane world volume.

The effective Lagrangians are found by calculating the pull back of the ten dimensional metric in (30) to the world volumes of the probes and expanding in powers of  $v^2$  [32]. Note that the world volumes are *not* the same. The D1/D5 probe, in static gauge, is aligned along the  $t, x^4$  directions while the F1/NS5 probe is aligned in the  $t, x^9$  plane.

We will use the D5-brane probe action in eqn. (28) for  $n_5$  D5-branes and include  $n_1$  D1-branes as instantons on the world volume so that  $q_1\tau_1 = (n_1 - n_5)\tau_1$  is the effective total D1-brane charge. We will also be working in static gauge where,

$$\xi^0 = t, \quad \xi^1 = x^4, \quad r = r(t, x^4), \quad \theta = \theta(t, x^4), \quad \phi = \phi(t, x^4) \quad (31)$$

and the probe is frozen on the K3 as well as the  $x^9$  circle. The kinetic term in the probe action (28) is found to be,

$$\mathcal{L} = \frac{1}{2} (n_5\tau_5 V h_1 + (n_1 - n_5)\tau_1 h_5) v^2 \quad (32)$$

where the ‘velocity’ is given by,

$$v^2 = h_{F1} h_{NS} \left[ \dot{r}^2 + r^2 \dot{\Omega}_2^2 - h_{F1}^{-1} h_{NS}^{-1} (r'^2 + r^2 \Omega_2'^2) \right] \quad (33)$$

here ‘dot’ and ‘prime’ denote differentiation with respect to  $t$  and  $x^4$  respectively. Also we have used the shorthand notation  $\dot{\Omega}_2^2 \equiv \dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2$  and likewise for  $\Omega_2'^2$ . The effective probe tension is given by the pre-factor in eqn. (32) and due to the relative sign appearing there it is expected to become negative at some radius for  $n_5 > n_1$ . Requiring that the tension always remain positive imposes that,

$$r \geq \frac{c_5 c_1}{n_5(c_5 - c_1) - n_1 c_1} (n_5(2N_5 - N_1) - n_1 N_5) \quad (34)$$

which, upon replacing  $n_5 \rightarrow \delta N_5$  and  $n_1 \rightarrow \delta N_1$ , is the *same* requirement found from the supergravity calculations. As pointed out in ref. [25, 28] this indicates that there is complete consistency between the probe and supergravity approaches. There are three special cases of this result. First, notice that setting  $n_5 = 0$  in these expressions, so that the probe is composed entirely of D1-branes, gives,

$$\mathcal{L} = \frac{1}{2} n_1 \tau_1 h_5 v^2 \quad (35)$$

and the effective tension is everywhere positive. This is simply the obvious result that D1-branes are not effected by compactification on a transverse K3 manifold and hence do not see any enhancement [33]. Next, setting  $n_1 = n_5$ , so that the D5-branes are ‘dressed’ with the appropriate number of instantons to effectively cancel the D1-brane charge induced by the wrapping on K3, eqn. (32) reduces to,

$$\mathcal{L} = \frac{1}{2} n_5 \tau_5 h_1 v^2 \quad (36)$$

which is also everywhere positive. This implies that a D5-brane probe can in fact move past the naively minimum radius in eqn. (34) and into the black hole so long as it is accompanied by the appropriate number of bound D1-branes. Finally, if there are no D1-branes bound to the D5-brane, *i.e.*,  $n_1 = 0$  then the D5-branes cannot access the region of the space time inside of,

$$\begin{aligned} r &= \frac{c_5 c_1}{c_5 - c_1} (2N_5 - N_1) \\ &= \frac{gl_s^2}{2R_9} \frac{(2N_5 - N_1)}{V/V^* - 1} = r_e \end{aligned} \quad (37)$$

which agrees exactly with the supergravity result in eqn. (19). As remarked above this is simply the enhançon radius originally found in ref. [21] rewritten in terms of parameters appropriate to four dimensions.

The above results generalize nicely to the NS5-brane probe. We choose static gauge so that the probe is aligned along the  $x^9$  direction, *i.e.*,

$$\zeta^0 = t, \quad \zeta^1 = x^9, \quad r = r(t, x^9), \quad \theta = \theta(t, x^9), \quad \phi = \phi(t, x^9) \quad (38)$$

as well as being frozen on the K3 and  $x^4$  circle. The probe action found in eqn. (29) for  $n_{NS}$  NS5-branes with  $n_{F1}$  bound F1-strings yields the following kinetic term,

$$\mathcal{L} = \frac{1}{2} (n_{NS} \tau_{NS} V h_{F1} + (n_{F1} - n_{NS}) \tau_{F1} h_{NS}) v^2 \quad (39)$$

where the ‘velocity’ is now given by,

$$v^2 = h_1 h_5 \left[ \dot{r}^2 + r^2 \dot{\Omega}_2^2 - h_1^{-1} h_5^{-1} (r'^2 + r^2 \Omega_2'^2) \right]. \quad (40)$$

The probe tension remains positive as long as,

$$r \geq \frac{c_{NS} c_{F1}}{n_{NS} (c_{NS} - c_{F1}) - n_{F1} c_{F1}} (n_{NS} (2N_{NS} - N_{F1}) - n_{F1} N_{NS}) \quad (41)$$

These results are, of course, the same as those for the D1/D5 probe above up to the exchange of D1/D5 labels for F1/NS5 labels. Again there are three interesting special cases. When  $n_{NS} = 0$  eqn. (39) collapses to the simple case of  $n_{F1}$  fundamental strings probing the black hole,

$$\mathcal{L} = \frac{1}{2} n_{F1} \tau_{F1} h_{NS} v^2. \quad (42)$$

The tension is everywhere positive and, just as for the solitary D1-branes above, these fundamental strings do not see any enhançon shell. When  $n_{F1} = n_{NS}$  so that the fundamental string charge, induced by wrapping the NS5-branes on the K3, is completely canceled,

$$\mathcal{L} = \frac{1}{2} n_{NS} \tau_{NS} h_{F1} v^2 \quad (43)$$

and the probe tension is again always positive. This ‘dressed’ NS5-brane does not see any enhançon and can thus be positioned at any arbitrary radius, in particular it can be moved

inside the black hole. Finally, if  $n_{F1} = 0$  then the NS5-brane probe cannot maintain positive tension unless it remains outside the enhançon radius,

$$\begin{aligned} r &= \frac{c_{NS}c_{F1}}{c_{NS} - c_{F1}} (2N_{NS} - N_{F1}) \\ &= \frac{g^2 l_s^2 (2N_{NS} - N_{F1})}{2R_4 (V/V^* - g^2)} = \tilde{r}_e \end{aligned} \quad (44)$$

Again this is in precise agreement with the supergravity calculations of the previous section.

In the next section we explore the implications of the results presented above for black hole thermodynamics.

## 4 Entropy and the Enhançon

### 4.1 Aspects of the Black Hole

To understand how the results obtained thus far are relevant for black hole physics it is instructive to attempt to build our black hole from its constituents. Consider a system of  $N_1, N_5, N_{F1}, N_{NS}$  D1-branes, D5-branes, F1-strings and NS5-branes respectively. To begin these are all assumed to be sitting at asymptotic infinity. Next, consider placing  $N'_1$  D1-branes and  $N'_{F1}$  F1-strings at the origin. In order to ensure that the local string coupling remains small in what follows we assume that  $N'_1 \ll N'_{F1}$ . The probe analysis of the previous section indicates that there is no obstacle to moving  $N'_5$  D5-branes and  $N'_{NS}$  NS5-branes to the origin to form a black hole so long as  $2N'_5 < N'_1$  and  $2N'_{NS} < N'_{F1}$ . The rest of the wrapped branes can come no closer than their respective enhançon shells located at  $r_e$  and  $\tilde{r}_e$  given in eqns. (19),(25). From this perspective it would seem that the limiting configuration, in which both of the enhançon shells sit exactly at the horizon, involves all of the available D1-branes and F1-strings while only utilizing  $N'_5 = N_1/2$  D5-branes and  $N'_{NS} = N_{F1}/2$  NS5-branes. The results of the probe calculations above—see eqns. (36),(43)—indicate however that when a D5-brane (NS5-brane) is suitably ‘dressed’ with a D1-brane (F1-string) that its tension will always be positive. Such a bound state is therefore free to proceed into the black hole. In this way one may construct a black hole with any values for the asymptotic charges by continually adding dressed D5- or NS5-branes until one either runs out of D5- or NS5-branes or all of the available D1-branes and F1-strings have been used up.

For each set of branes (D1/D5 or F1/NS5) there are three possibilities. As an illustration consider only the D1/D5-brane sector. The exact same comments will apply to the F1/NS5 sector. First, when  $0 < N_1 < N_5$  the black hole can only contain up to  $N'_5 = N_1$  D5-branes. The remaining D5-branes must remain at the enhançon radius since there are no more D1-branes with which to dress them. Second, if  $N_5 < N_1 < 2N_5$  then the black hole can absorb all of the D5-branes. In this case however there is still a region outside of the horizon where an additional D5-brane probe would have negative tension since from eqn. (19) the enhançon radius is still positive. Finally if  $2N_5 < N_1$  then all of the D5-branes can clearly be absorbed and there is no enhançon appearing outside of the horizon so that a D5-brane probe of the geometry set up by this final configuration could be brought from infinity and fall through the horizon without

encountering any difficulty associated with enhançons. Of course, all of the comments made here apply directly to the F1/NS5 components of this black hole as well.

The supergravity solution which describes the second and third scenarios is simply given by eqn. (1) taken in the range  $0 < r < \infty$ . This is because all of the available branes can be used in the construction of the black hole. In the first case the excess D5- or NS5-branes must reside at their respective enhançon radii outside the horizon. The solution therefore is given by a patched metric as discussed in section 2 where the ‘hatted’ harmonic functions, appearing in eqns. (10),(20), are used to describe the geometry interior to the enhançon loci while the ‘unhatted’ harmonic functions describe the geometry exterior to the enhançon loci. It should be clear that it is possible for both of the D5-brane and NS5-brane enhançon radii to be positive, or for both to be negative or a combination of the two. In other words all combinations of the hatted and unhatted harmonic functions are possible.

The conclusion to be drawn from this discussion is that enhançon physics is relevant for these black holes when the number of D1-branes (F1-strings) is small compared to the number of D5-branes (NS5-branes). In particular the enhançon mechanism places an upper limit on the number of D5-branes (NS5-branes) that can be used to build the black hole when there are a finite number of D1-branes (F1-strings) available. As was shown for five dimensional black holes in ref. [28] and as will be shown in the next section, this facet of enhançon physics is absolutely crucial for ensuring that this class of black holes are not in direct conflict with the second law of thermodynamics.

Finally, it should be emphasized that any excess D5-branes or NS5-branes that cannot be brought into the black hole, in the case  $0 < N_1 < N_5$  for example, can either be left on the enhançon shell or moved off to infinity and removed from the problem. The area of the horizon, and thus the entropy of the black hole, is identical in either case.

## 4.2 The Second Law and the Enhançon

The Bekenstein-Hawking entropy for the black holes considered here is given by,

$$S = \frac{A}{4G} = 2\pi \sqrt{(N_1 - N_5)(N_{F1} - N_{NS})N_5 N_{NS}}. \quad (45)$$

As in the case of five dimensional black holes[28] it is instructive to consider how this quantity depends on the numbers of D5-branes and NS5-branes for a fixed quantity of D1-branes and F1-strings. As can be seen in fig. (1) this is described by a half-ellipsoid. It is clear from this plot that in order to maximize the entropy the black hole should be constructed from precisely  $N_5 = N_1/2$  D5-branes and  $N_{NS} = N_{F1}/2$  NS5-branes. Intriguingly when there are only a small number of available D1-branes and/or fundamental strings the entropy is maximized by utilizing only a fraction of the available D5- or NS5-branes. We will comment on this further in the discussion section. For now however note that this maximum entropy black hole occurs exactly when *both* enhançon radii coincide with the horizon at  $r = 0$ .

To understand the connection between the second law and enhançon physics let us now revisit the construction of this black hole in the previous section. Recall that we began with a large, but fixed number of D1-branes and F1-strings inside the black hole. This places us somewhere near the origin (left corner) of fig. (1). Now since both  $N_1 > 2N_5$  and  $N_{F1} > 2N_{NS}$

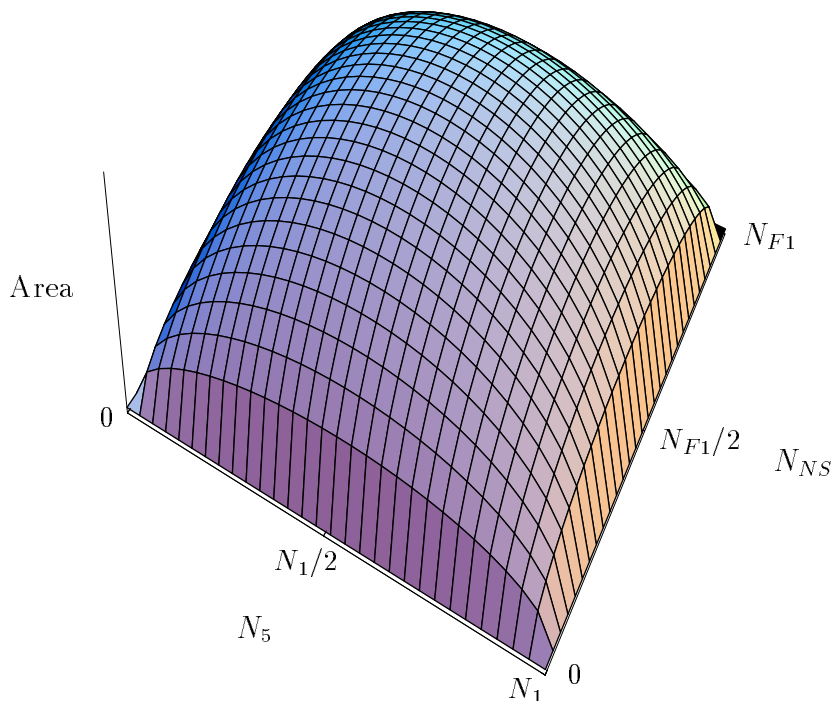


Figure 1: The area of the black hole horizon as a function of  $N_5$  and  $N_{NS}$  for fixed  $N_1$  and  $N_{F1}$ . Maximizing the area requires that  $N_5 = N_1/2$  and  $N_{NS} = N_{F1}/2$ .

are presumed to be satisfied we can add, freely, either D5-branes or NS5-branes and in the process climb towards the apside of the ellipsoid in fig. (1). Now suppose, for example, that we reach a point at which  $N_{NS} = N_{F1}/2$  while  $N_1 > 2N_5$  continues to be satisfied. We may either add another D5-brane, thereby continuing our progression towards the absolute maximum, or we may add another NS5-brane and *decrease* the entropy! It should be re-emphasized that this system is supersymmetric and additional D5- and NS5-branes may be brought in from infinity as slowly as one likes *i.e.*, adiabatically. Thus adding one more NS5-brane is clearly a violation of the second law of thermodynamics: Entropy can never decrease via such a process.

The resolution of this conundrum [28] is that precisely when  $N_{NS} = N_{F1}/2$  the enhançon radius for the NS5-branes is coincident with the event horizon and thus our attempt to add an additional NS5-brane into the fray is circumvented.

It is of course possible to bind NS5-branes with additional fundamental strings and then allow the bound state to cross the horizon. The enhançon also prevents probes of this type from lowering the entropy. Consider attempting to add a probe consisting of  $n_{F1}$  fundamental strings bound to  $n_{NS}$  NS5-branes. Assuming these to be small numbers compared with  $N_{F1}$  and  $N_{NS}$  it is easy to see from eqn. (45) that the resulting change in the (square of the) entropy will be,

$$\delta S^2 = 4\pi^2(N_1 - N_5)N_5 (n_{NS}(N_{F1} - 2N_{NS}) + n_{F1}N_{NS}) \quad (46)$$

which is negative precisely when the enhançon appears outside of the horizon in eqn. (41)! The

arguments of this section have focussed primarily on the F1/NS5 sector of these black holes. However, it should be clear that replacing NS5-branes by D5-branes and F1-strings by D1-branes in the preceding arguments shows that the enhançon prevents any attempts to violate the second law in this sector of the black hole as well.

To conclude, a naive interpretation of eqn. (45) indicates that by indiscriminately adding D5-branes or NS5-branes to this class of black holes one could easily violate the second law. As we have seen, any such attempt is strongly opposed by the enhançon.

## 5 Duals of the Enhançon in five dimensions

Motivated by the appearance (and necessity) of enhançon physics for NS5-branes in the context of four dimensional black holes we now study the emergence of the enhançon for NS5-branes in isolation. We do so for both the NS5-branes of type IIB and type IIA string theory.

In the original work of [21] the enhançon was unearthed by considering D6-branes wrapped on K3, or equivalently the T-dual set up of D5-brane wrapped on K3. Performing a type IIB S-duality transformation on the latter configuration yields a type IIB NS5-brane (NS5<sub>B</sub>) wrapped on K3. As argued elsewhere in this paper the negative D1-brane charge induced by wrapping the D5-branes on K3 transforms under S-duality to give negative fundamental string charge bound to the world volume of the NS5<sub>B</sub>-branes. As in the specific example of the black hole above this leads to an enhançon locus. This will be briefly demonstrated below via both supergravity and probe calculations.

One may also consider compactifying and performing a T-duality along the effective string which remains after the NS5<sub>B</sub>-branes have been wrapped on K3. This leaves NS5<sub>A</sub>-branes in type IIA string theory wrapped on a K3 manifold. The induced fundamental strings which were wound around the (compact) direction on the IIB side of this T-duality become induced momentum modes propagating on the T-dual circle. It is tempting to conjecture that this too will yield enhançon physics. Evidence, via a supergravity calculation, will be presented that this is indeed the case.

### 5.1 Type IIB NS5-branes and the Enhançon

We begin by considering the supergravity solution for the D1/D5 system of type IIB compactified on K3. The Einstein frame solution is,

$$\begin{aligned}
ds^2 &= h_1^{-3/4} h_5^{-1/4} (-dt^2 + dx_9^2) + h_1^{1/4} h_5^{3/4} (dr^2 + r^2 d\Omega_3^2) + h_1^{1/4} h_5^{-1/4} ds_{K3}^2 \\
e^{2\phi} &= \frac{h_1}{h_5} \\
C_{(2)} &= h_1^{-1} dt \wedge dx^9 \\
C_{(6)} &= h_5^{-1} dt \wedge dx^5 \cdots \wedge dx^9
\end{aligned} \tag{47}$$

where  $ds_{K3}^2$  is the metric on a K3 surface with volume  $V$  on which the directions  $x^5, x^6, x^7, x^8$  are wrapped. For later purposes  $x^9$  is taken to be periodic with period  $2\pi R_9$ . Also,  $h_1$  and



$h_5$  are harmonic functions appropriate to five transverse directions. Performing an S-duality transformation leaves us in type IIB however the solution (47) now becomes that of an NS5-brane bound to a fundamental string. The Einstein frame solution for such an F1/NS5 bound state in type IIB wrapped on a K3 surface is given by,

$$ds^2 = h_{F1}^{-3/4} h_{NS}^{-1/4} (-dt^2 + dx_9^2) + h_{F1}^{1/4} h_{NS}^{3/4} (dr^2 + r^2 d\Omega_3^2) + h_{F1}^{1/4} h_{NS}^{-1/4} ds_{K3}^2 \quad (48)$$

The dilaton and Kalb-Ramond fields are given by,

$$\begin{aligned} e^{2\phi} &= \frac{h_{NS}}{h_{F1}} \\ B_{(2)} &= h_{F1}^{-1} dt \wedge dx^9 \\ B_{(6)} &= h_{NS}^{-1} dt \wedge dx^5 \cdots \wedge dx^9 \end{aligned} \quad (49)$$

In these expressions the harmonic functions are,

$$h_{F1} = 1 + \frac{c_{F1} Q_{F1}}{r^2} \quad h_{NS} = 1 + \frac{c_{NS} Q_{NS}}{r^2}. \quad (50)$$

As in the previous sections we place  $\delta N_{F1}$  fundamental strings and  $\delta N_{NS}$  NS5-branes on a shell located at some radius  $R_c$  so that the interior geometry is given by the metric in eqn. (48) using the following ‘hatted’ functions,

$$\hat{h}_{F1} = 1 + \frac{c_{F1} Q'_{F1}}{r^2} + \frac{c_{F1} (Q_{F1} - Q'_{F1})}{R_c^2} \quad \hat{h}_{NS} = 1 + \frac{c_{NS} Q'_{NS}}{r^2} + \frac{c_{NS} (Q_{NS} - Q'_{NS})}{R_c^2} \quad (51)$$

The  $c_i$  are now the relevant five dimensional quantities [12],

$$c_{F1} = g^2 l_s^2 \frac{V^\star}{V} \quad c_{NS} = l_s^2 \quad (52)$$

Explicit calculation determines the stress tensor at the junction surface  $r = R_c$  to be,

$$\begin{aligned} 8\pi G S_{\mu\nu} &= \frac{\sigma}{2} \left( \frac{h'_{NS}}{h_{NS}} + \frac{h'_{F1}}{h_{F1}} - \frac{\hat{h}'_{NS}}{\hat{h}_{NS}} - \frac{\hat{h}'_{F1}}{\hat{h}_{F1}} \right) g_{\mu\nu} \\ 8\pi G S_{ab} &= \frac{\sigma}{2} \left( \frac{h'_{NS}}{h_{NS}} - \frac{\hat{h}'_{NS}}{\hat{h}_{NS}} \right) g_{ab} \\ 8\pi G S_{ij} &= 0 \end{aligned} \quad (53)$$

where  $\mu, \nu$  are the  $t, x_9$  directions,  $a, b$  are the K3 directions and  $i, j$  are the three-sphere directions. In this expression  $S_{\mu\nu}$  has the interpretation as the stress-energy in the effective string lying in the  $t, x^9$  plane.  $S_{ab}$  is the stress-energy in the K3 directions which here is, as expected, coming entirely from the NS5-branes. The stress-energy in the three-sphere directions vanishes as it should since this is a BPS configuration. The tension of the effective string lying in the  $x^9$  directions can again —see section 2— be put in the form expected from varying the Einstein frame world volume actions,

$$T = \frac{h_{F1}^{-1/4} h_{NS}^{1/4} (Q_{F1} - Q'_{F1}) \tau_{F1}}{A_{S^3} A_{K3}} + \frac{h_{F1}^{1/4} h_{NS}^{-1/4} (Q_{NS} - Q'_{NS}) \tau_{NS}}{A_{S^3}} \quad (54)$$

where  $Q_{F1} - Q'_{F1} = \delta N_{F1} - \delta N_{NS}$  and  $Q_{NS} - Q'_{NS} = \delta N_{NS}$ . Substituting the harmonic functions in eqn. (50) this expression is non-negative only for,

$$R_c^2 \geq \frac{c_{NS}c_{F1}}{\delta N_{NS}(c_{NS} - c_{F1}) - \delta N_{F1}c_{F1}} (\delta N_{NS}(2N_{NS} - N_{F1}) - \delta N_{F1}N_{NS}) \quad (55)$$

which for  $\delta N_{F1} = 0$  can be evaluated as,

$$\begin{aligned} R_c^2 &\geq \frac{c_{NS}c_{F1}}{c_{NS} - c_{F1}} (2N_{NS} - N_{F1}) \\ &= \frac{g^2 l_s^2}{V/V^* - g^2} (2N_{NS} - N_{F1}) \equiv R_e^2. \end{aligned} \quad (56)$$

Note that up to the standard S-duality transformations,  $g \rightarrow 1/g$  and  $l_s^2 \rightarrow gl_s^2$ , this is exactly the enhançon locus found in ref. [21]. The point is that S-duality, and more generally U-duality simply rotate the charges and the constants  $c_i$  into each other.

One can also see the same result emerging by performing a probe calculation using the action in eqn. (29). Once again consider a probe composed of  $n_{F1}$  F1-strings which are bound to  $n_5$  NS5-branes. Choosing ‘static gauge’ we align the coordinates  $\zeta^1, \zeta^2$  of the effective string with the  $t, x^9$  direction and allow it to move in the four non-compact directions. Note that it is frozen on the K3. Explicitly calculating the pull-back of eqn. (48) we find that the mass and charge cancel as required by supersymmetry and the effective Lagrangian can be written,

$$\mathcal{L} = \frac{1}{2} (n_{NS}\tau_{NS}Vh_{F1} + (n_{F1} - n_{NS})\tau_{F1}h_{NS})v^2 \quad (57)$$

where the ‘velocity’ is now given by,

$$v^2 = [\dot{r}^2 + r^2\dot{\Omega}_3^2 - r'^2 - r^2\Omega_3'^2]. \quad (58)$$

and the probe tension is physical only for radii satisfying eqn. (55).

As a final comment we point out that unlike the D1/D5 version of this system the enhançon radius is not associated with any special behavior of the K3 surface. In fact the (string frame) volume of the K3 is a constant in this metric! Note, however that  $R_c$  is the point at which the local value of the string coupling becomes,

$$g^2 e^{2\phi} = g^2 h_{NS}/h_{F1} = V/V^* \quad (59)$$

which is generically a large number. One might therefore worry that string loops will strongly alter the picture presented here and perhaps negate the appearance of the enhançon within this system. There are at least two reasons to believe this will not be the case. First, as was demonstrated in section 3, U-duality of type II string theory allows for the construction of a black hole which, without enhançon physics for NS5-brane’s, could easily be used to violate the second law of thermodynamics. Further, recall that in this construction the local dilaton could always be kept small and hence string loops could be neglected. Second, as was pointed out in ref. [21], within the enhançon locus is a region of enhanced gauge symmetry. It seems unlikely that this extra gauge symmetry would somehow be destroyed at strong coupling. Therefore it ought to be present in any S-dual formulation. The fact that the enhançon radii for the D1/D5 system and the F1/NS5<sub>B</sub> systems are simply S-dual to each other seems to support this point of view.

## 5.2 Type IIA NS5-branes and the Enhançon

In this subsection we analyze the emergence of the enhançon for the case of NS5<sub>A</sub> branes in type IIA. Consider performing a T-duality transformation<sup>3</sup> on the  $x^9$  direction in the metric (48). This yields an NS5<sub>A</sub>-brane bound to momentum propagating in the  $x^9$  direction in type IIA string theory. It should be emphasized that this momentum is induced by the curvature of K3. The Einstein frame metric is,

$$ds^2 = h_5^{-1/4} \left( -dt^2 + dx_5^2 + k(dt - dx_5)^2 \right) + h_5^{3/4} \left( dr^2 + r^2 d\Omega_3^2 \right) + h_5^{-1/4} ds_{K3}^2 \quad (60)$$

while the dilaton and Kalb-Ramond fields are,

$$e^{2\phi} = h_{NS} \quad B_{(6)} = h_{NS}^{-1} dt \wedge \cdots \wedge dx^9 \quad (61)$$

in this expression  $h_{NS}$  is identical to that in eqn. (50) while the ‘momentum’ function is,

$$k = \frac{c_p Q_p}{r^2} \quad c_p = \frac{g^2 l_s^4}{R_9^2} V^* / V \quad (62)$$

Recall that  $Q_p = N_p - N_{NS}$  and  $N_{NS} = Q_{NS}$  *i.e.*, we have explicitly included  $N_p$  momentum modes propagating along the effective string in addition to those induced by the curvature. As in the previous cases we place  $\delta N_p$  momentum modes and  $\delta N_{NS}$  NS5-branes on a shell located at  $r = R_b$ . Inside of the shell the harmonic function for the momentum becomes,

$$\hat{k} = \frac{c_p Q'_p}{r^2} + \frac{c_p (Q_p - Q'_p)}{R_b^2}. \quad (63)$$

The harmonic function for the NS5-branes was already given in eqn. (51). Calculating the stress tensor of the junction surface yields a more complicated expression than before which can be put in the form,

$$\begin{aligned} 8\pi G S_{\mu\nu} &= \frac{\sigma}{2} \left( \frac{h'_{NS}}{h_{NS}} - \frac{\hat{h}'_{NS}}{\hat{h}_{NS}} \right) g_{\mu\nu} + \frac{\sigma}{2} (\hat{k}' - k') p_\mu p_\nu \\ 8\pi G S_{ab} &= \frac{\sigma}{2} \left( \frac{h'_{NS}}{h_{NS}} - \frac{\hat{h}'_{NS}}{\hat{h}_{NS}} \right) g_{ab} \\ 8\pi G S_{ij} &= 0 \end{aligned} \quad (64)$$

where  $\mu, \nu$  span  $t, x^9$  while  $a, b$  are indices on the K3 and  $i, j$  represent the angles of the three-sphere at  $r = R_b$ . Here we have also defined the vector  $\mathbf{p} = h_{NS}^{-1/8} (-1, 1, \mathbf{0})$ . Note that this is a null vector. The  $S_{\mu\nu}$  component is just saying that there is a string with a tension given by the piece proportional to the metric with a momentum wave traveling at the speed of light along it. This is an intuitively pleasing answer but it is also problematic. Thus far the procedure has been to determine at which radius the tension of an effective string vanishes. The vanishing occurs because of competing terms in the total tension of the string. Usually one does not think about balancing the tension of an extended object with momentum to get a zero result, it would seem however that this is what is required in this case. In fact one can recover the expected enhançon radius by setting the energy density,  $S_{tt} = 0$ . In order to alleviate any anxiety regarding this

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<sup>3</sup>Under T-duality  $R \rightarrow l_s^2/R$  and  $g \rightarrow gl_s/R$  when T-duality is performed on a circle of radius  $R$ .

procedure recall that above it was alluded to that the constants appearing in the enhançon radius are just mixed up by U-duality. This is the five dimensional U-duality group. We will therefore perform this analysis in five dimensions where all of the charges are interpreted as point like sources and are therefore on equal footing. To do so we reconsider this bound state in a dimensionally reduced set-up. The string frame metric in a form suitable for dimensional reduction is,

$$ds^2 = -\frac{dt^2}{1+k} + (1+k) \left( dx_9 - \frac{k}{1+k} dt \right)^2 + h_{NS} (dr^2 + r^2 d\Omega_3^2) + ds_{K3}^2 \quad (65)$$

We perform the dimensional reduction on this and shift to the five-dimensional Einstein frame using,  $g_{AB}^{Einstein} = e^{-4\phi_5/3} g_{AB}^{string}$  where the five dimensional dilaton is given by,

$$e^{2\phi_5} = (2\pi R_9 V)^{-1} (1+k)^{-1/2} h_{NS}. \quad (66)$$

The dimensionally reduced Einstein frame metric is,

$$ds_5^2 = -(h_{NS}(1+k))^{-2/3} dt^2 + (h_{NS}(1+k))^{1/3} (dr^2 + r^2 d\Omega_3^2) \quad (67)$$

Applying the same analysis as above we suppose that  $\delta N_{NS}$  NS5-branes and  $\delta N_p$  momentum modes get held up at the radius  $R_b$  and imagine matching onto the interior solution with the hatted functions as above. Calculating the stress tensor at the surface  $r = R_b$  now gives,

$$\begin{aligned} 8\pi G S_{tt} &= \frac{\sigma}{2} \left( \frac{h'_{NS}}{h_{NS}} - \frac{\hat{h}'_{NS}}{\hat{h}_{NS}} + \frac{k'}{1+k} - \frac{\hat{k}'}{1+\hat{k}} \right) g_{tt} \\ 8\pi G S_{ij} &= 0 \end{aligned} \quad (68)$$

The energy density is now simply the negative of the factor multiplying  $g_{tt}$ ,

$$E = \frac{\sigma}{2} \left( \frac{\hat{k}'}{1+\hat{k}} - \frac{k'}{1+k} + \frac{\hat{h}'_{NS}}{\hat{h}_{NS}} - \frac{h'_{NS}}{h_{NS}} \right). \quad (69)$$

Substituting the harmonic functions the energy density is non-negative only for,

$$R_b^2 \geq \frac{c_{NS} c_P}{\delta N_{NS} (c_{NS} - c_P) - \delta N_P c_P} (\delta N_{NS} (2N_{NS} - N_P) - \delta N_P N_{NS}) \quad (70)$$

and assuming that  $\delta N_p = 0$

$$\begin{aligned} R_b^2 &\geq \frac{c_{NS} c_P}{c_{NS} - c_P} (2N_{NS} - N_p) \\ &= g^2 l_s^2 \frac{2N_{NS} - N_p}{\frac{R_a^2}{l_s^2} V / V^* - g^2} \equiv \tilde{R}_e^2 \end{aligned} \quad (71)$$

and thus there is an enhançon like effect occurring here as well. One may note that under the standard T-duality transformations this becomes precisely the expression found above for the NS5<sub>B</sub>-brane in eqn. (57).

## 6 Discussion

At strong coupling, or in large numbers, Dp-branes produce a non-trivial back reaction on the geometry and their effective description lies within the realm of classical supergravity. Therefore the appearance of naked singularities and violations of the laws of black hole mechanics in the gravitational description of Dp-branes wrapped on K3 manifolds would both pose serious problems. On the one hand the occurrence of naked singularities casts doubt on whether or not supergravity is in fact a consistent description of low energy string theory compactified on K3. On the other hand violations of the second law of thermodynamics by collections of BPS objects in string theory would be a major obstacle for string theory itself. Happily both of these undesirable scenarios are prevented by the enhançon mechanism. It seems therefore that the enhançon is itself a fundamental aspect of string theory compactifications and one wonders if its influence is not more ubiquitous.

Black holes have always provided an interesting arena in which to explore and gain an understanding of fundamental aspects of quantum gravity and string theory. In this paper we have seen how the enhançon mechanism of ref. [21] plays a central role in ensuring that four dimensional black holes embedded into K3 compactifications of type IIB string theory are consistent with the second law of thermodynamics.

Especially crucial to this result is the fact, implied by S-duality, that NS5-branes in K3 compactifications are subject to the enhançon mechanism. Further T-duality, and therefore U-duality, implies that the same physics is relevant for type IIA NS5-branes as well. In fact since all of the enhançon radii uncovered here were dependent only on the constants generically labelled  $c_i$  where  $i = 1, 5, F1, NS, P$  it seems that the U-duality *covariant* expression for the enhançon radius, analogous to eqns. (19),(25),(57),(71) for any stringy object which entirely wraps a K3 manifold can be written in  $d$ -dimensions as,

$$r_e^{d-3} = \frac{c_A c_b}{c_A - c_b} (2N_A - N_b). \quad (72)$$

Here the upper case indices represent the constant associated with the wrapped object and the lower case indices that of the charged object induced by the curvature of K3. Establishing the validity of this formula in general would be very interesting. One immediate implication would be that enhançon physics is important for Kaluza-Klein (KK) monopoles[34]. Another indication that this is the case is that U-duality guarantees that the charges making up the black hole considered in this paper can be rotated to produce a black hole constructed out of D1-branes, D5-branes, KK monopoles and momentum modes. The microscopic entropy of this black hole was considered in refs.[8, 11]. If this black hole were to avoid violating the second law it too would be subject to enhançon physics for *both* the D5-branes as well as the KK monopoles.

Enhançon physics for objects other than D-branes raises some curious puzzles. Recall that in the case of a Dp-brane wrapped on K3 that there is one unit of negative charge corresponding to a D(p-4)-brane induced onto the world volume. This does not, however, correspond to anti-D(p-4)-brane charge. The induced charge preserves the same supersymmetries as a regular D(p-4)-brane. To be specific consider the D1/D5 system. In the discussions above  $N_1$  D1-branes were bound to  $N_5$  D5-branes which were wrapped on K3 inducing  $-N_5$  units of D1-brane charge. The induced charge is aligned with the *same* orientation as the ‘real’ D1-branes.

Under S-duality this becomes the F1/NS5 bound state of type IIB string theory with  $-N_{NS}$  units of curvature induced F1-string charge, again aligned parallel to the ‘real’ F1-strings. As discussed above T-duality takes this to  $N_{NS}$  NS5-branes in IIA bound to  $N_p$  units of right moving momentum and  $-N_{NS}$  units of induced momentum which are also right moving. Here the induced momentum **cannot** be interpreted as left moving momentum in the same way as the induced D1-branes that we began with are not anti-D1-branes. The induced momentum preserves the same supersymmetries as the right moving momentum and therefore is itself right moving. To see this more clearly note that from eqns. (60) and (62) the  $t, x^5$  components of the metric are,

$$ds^2 \propto -dt^2 + dx_5^2 + \frac{c_p(N_p - N_{NS})}{r^2}(dt - dx_5)^2. \quad (73)$$

The fact that it is  $dt - dx_5$  which appears and not  $dt + dx_5$  indicates that this is all right moving momentum, even though there are  $N_p - N_{NS}$  units of it. It seems difficult to understand how this might be incorporated into the microscopic entropy of a black hole constructed from a D2/D6/P/NS5 bound state wrapped on K3. Usually the contribution of the momentum modes to the degeneracy of states is accounted for in a partition function. It seems that one would partition only the  $N_p$  units of ‘real’ momentum since the NS5-brane’s contribution enters in the  $4Q_2Q_6Q_{NS}$  massless (2, 6) strings which carry the momentum modes. Of course this gives the wrong answer since the asymptotic charge which must appear in the entropy formula is  $Q_p = N_p - N_{NS}$ . The same comments clearly apply to the D1/D5/F1/NS5 black hole considered in this paper with ‘momentum’ replaced by ‘winding’. It would be interesting to understand how the microscopic counting works out in detail.

The fact that it is entropically favoured for black holes to form which are made up of precisely  $N_5 = N_1/2$  D5-branes and  $N_{NS} = N_{F1}/2$  NS5-branes is a very interesting result that certainly deserves more investigation. U-duality suggests that these relations should be true no matter which four charges make up the black hole and it would be very useful if one could express these as invariants of the four dimensional U-duality group. Alternatively, insight as to why these particular values of the charges are favoured might be gained from studying slightly non-extremal versions of these black holes. Non-extremality would lift the degeneracy of the supersymmetric vacua and the dynamics would pick a preferred ground state. As discussed in ref. [28] this leads to the intriguing possibility that D5-branes and NS5-branes can be expelled from these black holes.

Another issue which warrants consideration is whether or not the enhançon enters into the attractor flow descriptions of black holes in compactified supergravity [35]. It seems probable that recent work of Denef and collaborators [36] is relevant to understanding this issue.

Finally it would be very interesting to understand the role played by the enhançon behind the horizon of our black holes. One might hope that there would be some sort of resolution of the time like singularity living there however as in the case of the five dimensional black holes [28] this does not seem to be the case.

The physics of the enhançon is clearly a rich and very interesting topic for investigation. Above and beyond the applications to four dimensional black holes presented in this paper and those mentioned briefly in the present discussion there are sure to be many interesting avenues of pursuit.

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